

Channel Coding

Selected Exercises

Question 1 For a (6, 3) systematic linear block code, the three parity-check digits are p_4 , p_5 and p_6 given by

$$p_4 = m_1 \oplus m_2 \oplus m_3$$

$$p_5 = m_1 \oplus m_2$$

$$p_6 = m_1 \oplus m_3$$

- (a) What is the generator matrix of this code?
- (b) What is the code generated by this code?
- (c) What will be the error detection/correction capability of this code?

Question 2 List all the code words for (7,3) block code for a given \mathbf{P} matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Also determine the hamming weight and hamming distance of the code.

Question-3 Consider a (7,4) code whose generator matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- (a) Find all codewords of the code.
- (b) Find \mathbf{H} , the parity-check matrix of the code.
- (c) Compute the syndrome for the received vector 1 1 0 1 1 0 1. Is this a valid code vector?
- (d) What is the error correcting capability of the code?
- (e) What is the error detecting capability of the code?

Question 4 Consider a systematic block code whose parity-check equations are

$$p_1 = m_1 \oplus m_2 \oplus m_4$$

$$p_2 = m_1 \oplus m_3 \oplus m_4$$

$$p_3 = m_1 \oplus m_2 \oplus m_3$$

$$p_4 = m_2 \oplus m_3 \oplus m_4$$

where m_i are the message digits and p_i are check digits.

- (a) Find the generator matrix and parity-check matrix for this code.
- (b) How many errors can the code correct?
- (c) Is the vector 10101010 a codeword?
- (d) is the vector 01011100 a codeword?

Question 5 Generator matrix (7,4) block code is

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

- (a) Determine the parity-check matrix.
- (b) Determine the maximum weight of the code.

Question 6 Design a $(n,k)=(5,2)$ linear block code

- (a) Choose the codeword to be in systematic form, and choose them with the goal of maximizing d_{\min} .
- (b) Find the generator matrix for the codeword set.
- (c) Calculate the parity-check matrix.
- (d) What are the error-correcting and error-detecting capabilities of the code
- (e) Make the syndrome table for the correctable error pattern

Question 7 Consider the (5,1) repetition code, which consists of the two codewords 00000 and 11111, corresponding to message 0 and 1, respectively. Is this a perfect code?